

Matthie Recalculation Path

Algebra: A Complete Course (Book 2)

Teacher Guide & Keys

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Teacher Guide & Keys

This guide covers every skill in the Algebra skill map (21 skills across Chapters 1-10). Each skill entry gives the mathematical claim, a Route Finder item, a Tripwire item that discriminates the skill's dominant confusion, a Confirmation item in a different surface form, a Recalculation Check, and a Road Test that checks transfer without naming the chapter: plus the full teacher pathway: observed response, likely reasoning (a hypothesis, not a diagnosis), how to confirm it, the teaching move, and where to route next.

A single wrong answer is a hypothesis, not a diagnosis. Confirm with a second independent piece of evidence before treating a pattern as established.

Teacher Desk Card

Keep this page open while working through any skill below. Four rules, one per stage of the cycle:

FIND: STOP FINDING when the evidence is sufficient to choose a teaching target. Do not measure everything the student cannot do.

TEACH: STOP RETEACHING the wider unit when the confirmed gap is narrower. Address the smallest idea actually blocking progress.

CHECK: STOP TESTING when the student has provided sufficient independent evidence. A fresh problem or existing classwork both count.

MOVE: STOP WORKING THE OLD TARGET once the student has met the defined mastery criterion. Record status and route to what's next.

Status codes: NY (Not Yet), R (Recalculating), M (Mastered), IE (Insufficient Evidence). Fixed across the whole series.

The lightest-evidence rule: do not administer a separate diagnostic form when reliable, current classwork already answers the question.

Quick Answer Key (Student Forms)

For fast grading against a completed Student Form. Organized in the same order as Student Forms: Route Finder, Tripwire, Confirmation, Road Test, per skill. For the reasoning, the reteach move, and where to route a student next, see that skill's full entry later in this guide.

Form ID	Item	Correct response
AL-EX-01-RF1	Route Finder	$3x + 5$ (already simplified: $3x$ and 5 are not like terms).
AL-EX-01-RF1	Tripwire	$4x$ ($x = 1x$, so $1x + 3x = 4x$).
AL-EX-01-CF1	Confirmation	No: $9y - y = 8y$ ($y = 1y$, so $9y - 1y = 8y$). The variable does not disappear.
AL-EX-01-CF1	Road Test	$7c$.
AL-EX-02-RF1	Route Finder	$8 - x$ ($5 - x + 3 = 8 - x$).
AL-EX-02-RF1	Tripwire	$10 - 3x$ ($8 - 3x + 2 = 10 - 3x$).
AL-EX-02-CF1	Confirmation	$19 - 5x$; at $x=1$: original = $12 - (5 - 7) = 12 - (-2) = 14$; simplified = $19 - 5 = 14$. Confirmed.
AL-EX-02-CF1	Road Test	$24 - 6x$ ($20 - (6x - 4) = 20 - 6x + 4 = 24 - 6x$).
AL-EX-03-RF1	Route Finder	No: it's an expression with no equals sign; it can only be simplified or evaluated at a given x .
AL-EX-03-RF1	Tripwire	$n - 5$ ('less than' means take the first number away from the second).
AL-EX-03-CF1	Confirmation	$3x - 5 = 10$ is an equation and can be solved ($x=5$). $3x - 5$ alone is an expression and can only be simplified or evaluated.
AL-EX-03-CF1	Road Test	$2n - 7$ (not $7 - 2n$).
AL-EX-04-RF1	Route Finder	14 ($3 \times 4 + 2 = 12 + 2 = 14$).
AL-EX-04-RF1	Tripwire	25 ($(-5)^2 = 25$: the entire value -5 is squared).
AL-EX-04-CF1	Confirmation	12 ($3 \times (-2)^2 = 3 \times 4 = 12$).
AL-EX-04-CF1	Road Test	-4 ($5 - (-3)^2 = 5 - 9 = -4$).

Form ID	Item	Correct response
AL-EX-05-RF1	Route Finder	7, and 21: same expression, different inputs, different outputs.
AL-EX-05-RF1	Tripwire	\$35 (or whatever unit the expression represents): the numeric result must be restated in context.
AL-EX-05-CF1	Confirmation	\$24 and \$44: different inputs give different outputs; the expression's value is not fixed.
AL-EX-05-CF1	Road Test	\$12.50 ($10 + 0.05 \times 50 = 10 + 2.50$), stated in dollars.
AL-EQ-01-RF1	Route Finder	$x = 7$ (subtract 5 from both sides).
AL-EQ-01-RF1	Tripwire	$x = -4$ (divide both sides by -3, not 3).
AL-EQ-01-CF1	Confirmation	$x = 3$; divide both sides by -5.
AL-EQ-01-CF1	Road Test	$w = 5$ kg.
AL-EQ-02-RF1	Route Finder	$23 - 8 = 15$. Confirmed.
AL-EQ-02-RF1	Tripwire	Check: $-3(-4) = 12$. Confirmed. The check step should not be skipped, even when the sign work looked difficult.
AL-EQ-02-CF1	Confirmation	Checking should still happen: $2 \times (-9) = -18$. Confirmed correct here, but the habit of skipping checks is risky regardless of how easy a problem feels.
AL-EQ-02-CF1	Road Test	Yes: $6 - 10 = -4$, confirmed by substitution.
AL-EQ-03-RF1	Route Finder	$x = 9$ (either distribute: $2x - 8 = 10$, $2x = 18$, $x = 9$; or divide by 2 first: $x - 4 = 5$, $x = 9$: both valid).
AL-EQ-03-RF1	Tripwire	$2x - 8 = 10$, so $x = 9$.

Form ID	Item	Correct response
AL-EQ-03-CF1	Confirmation	$x = 7$ both ways: distribute: $4x-8=20$, $4x=28$, $x=7$; divide first: $x-2=5$, $x=7$.
AL-EQ-03-CF1	Road Test	$x = 8$ (either method).
AL-EQ-04-RF1	Route Finder	Infinitely many solutions (the statement $3 = 3$ is always true).
AL-EQ-04-RF1	Tripwire	No mistake: $2 = -3$ is false, which means the equation has no solution.
AL-EQ-04-CF1	Confirmation	$-2 = 3$, false: no solution.
AL-EQ-04-CF1	Road Test	Always: the two expressions are identical, so every value of m works (infinitely many solutions).
AL-IN-01-RF1	Route Finder	$x < -4$ (divide by -2 , flip the sign).
AL-IN-01-RF1	Tripwire	$x < -5$ (dividing by -4 requires flipping the sign).
AL-IN-01-CF1	Confirmation	$x \leq 5$; dividing by a negative flips the sign from $\geq$ to $\leq$.
AL-IN-01-CF1	Road Test	$h \geq 5$ (dividing by -2 flips the sign).
AL-IN-02-RF1	Route Finder	Closed circle at 5 , arrow pointing right (values 5 and greater).
AL-IN-02-RF1	Tripwire	Open circle at -2 , arrow pointing left (values less than -2 are to the left on a number line).
AL-IN-02-CF1	Confirmation	Closed circle at -3 , arrow left; -3 and -4 are included, -2 is not.
AL-IN-02-CF1	Road Test	Closed circle at 6 , arrow pointing left (values 6 and below).
AL-LN-01-RF1	Route Finder	$m = 3$, $b = -2$.
AL-LN-01-RF1	Tripwire	$m = -1$ (since $-x = -1x$), $b = 4$.
AL-LN-01-CF1	Confirmation	$m = 1$, $b = -9$.

Form ID	Item	Correct response
AL-LN-01-CF1	Road Test	Decreases by 1 per unit (slope -1); starts at 50 (y-intercept 50).
AL-LN-02-RF1	Route Finder	$m = 2 \left(\frac{10-2}{5-1} = \frac{8}{4} = 2 \right)$.
AL-LN-02-RF1	Tripwire	The horizontal line has slope 0; the vertical line has undefined slope (the run is 0, and division by 0 is undefined).
AL-LN-02-CF1	Confirmation	Undefined slope (run = 0); this is a vertical line.
AL-LN-02-CF1	Road Test	First: undefined slope (vertical). Second: slope 0 (horizontal).
AL-LN-03-RF1	Route Finder	$y - 5 = 4(x - 1)$.
AL-LN-03-RF1	Tripwire	$y + 3 = 5(x - 2)$, since $y - (-3) = y + 3$.
AL-LN-03-CF1	Confirmation	$y + 5 = -3(x + 2)$; simplifies to $y = -3x - 11$.
AL-LN-03-CF1	Road Test	$y + 1 = 2(x - 5)$, which simplifies to $y = 2x - 11$.
AL-LN-04-RF1	Route Finder	$-\frac{1}{3}$ (negative reciprocal).
AL-LN-04-RF1	Tripwire	$-\frac{1}{4}$ (the negative reciprocal, since $4 \times (-\frac{1}{4}) = -1$).
AL-LN-04-CF1	Confirmation	$\frac{6}{5}$; check: $(-\frac{5}{6}) \times (\frac{6}{5}) = -1$. Confirmed.
AL-LN-04-CF1	Road Test	$-\frac{1}{3}$.
AL-SY-01-RF1	Route Finder	$x = 4, y = 1$ ($2x - (x-3) = 7, x + 3 = 7, x = 4, y = 1$).
AL-SY-01-RF1	Tripwire	$2x - 4(x - 3) = 10$, which becomes $2x - 4x + 12 = 10$.
AL-SY-01-CF1	Confirmation	$x = 3, y = 2$.
AL-SY-01-CF1	Road Test	$x = 17, y = 13$.
AL-SY-02-RF1	Route Finder	(5, 2): an ordered pair, not just $x = 5$.

Form ID	Item	Correct response
AL-SY-02-RF1	Tripwire	No mistake: $14 = 15$ is false, meaning the system has no solution (the lines are parallel).
AL-SY-02-CF1	Confirmation	Substituting shows $8 = 9$, false: no solution (parallel lines).
AL-SY-02-CF1	Road Test	True statement: infinitely many solutions; the two plans are identical.
AL-PL-01-RF1	Route Finder	$3x^2 + 2x$.
AL-PL-01-RF1	Tripwire	$3x^2 - 8x$ (distribute the negative across both terms: $4x^2 - 3x - x^2 - 5x = 3x^2 - 8x$).
AL-PL-01-CF1	Confirmation	$-3x^2 + 2x + 8$.
AL-PL-01-CF1	Road Test	Any correctly distributed and combined result, provided the negative is applied to every term of the second polynomial.
AL-PL-02-RF1	Route Finder	$x^2 + 7x + 12$.
AL-PL-02-RF1	Tripwire	$x^2 + 10x + 25$ ($(x+5)(x+5) = x^2 + 5x + 5x + 25$).
AL-PL-02-CF1	Confirmation	$x^2 + 5x - 24$.
AL-PL-02-CF1	Road Test	$x^2 + 8x + 16$.
AL-PL-03-RF1	Route Finder	$2(x + 2)(x + 3)$ (GCF of 2 first, then factor the trinomial).
AL-PL-03-RF1	Tripwire	$2(x + 2)(x + 3)$: the trinomial factoring is correct, but the GCF of 2 was never factored out first.
AL-PL-03-CF1	Confirmation	$4(x + 2)(x + 3)$.
AL-PL-03-CF1	Road Test	$5(x + 2)(x + 3)$.
AL-PL-04-RF1	Route Finder	$(x + 4)(x - 4)$; expanding gives $x^2 - 16$, confirmed.

Form ID	Item	Correct response
AL-PL-04-RF1	Tripwire	$x^2 + 25$ does not factor over the reals: $(x+5)(x-5)$ expands to $x^2 - 25$, not $x^2 + 25$, so the pattern doesn't apply to a sum.
AL-PL-04-CF1	Confirmation	$(x+10)(x-10)$, confirmed by expanding. $x^2 + 100$ cannot use this pattern since it is a sum, not a difference.
AL-PL-04-CF1	Road Test	Only $x^2 - 36$ factors this way, as $(x+6)(x-6)$; $x^2 + 36$ does not factor over the reals using this pattern.

Chapter 1: Introduction to Algebra

AL-EX-01 Combine like terms only

Mathematical claim: Terms combine only if they share the same variable and exponent; a bare variable's coefficient is 1, not 0 or absent.

Route Finder

Item: Simplify: $3x + 5$

Correct response: $3x + 5$ (already simplified: $3x$ and 5 are not like terms).

Tripwire

Observed: A student simplifies $x + 3x$ and writes $3x$.

Likely wrong response: $3x$, from treating the bare x as having no coefficient ($0x$) rather than $1x$.

Correct response: $4x$ ($x = 1x$, so $1x + 3x = 4x$).

Likely reasoning: may indicate a bare variable is being read as having no coefficient, rather than an implied coefficient of 1

Confirm (approach): Give a second item combining a bare variable with a like term, in a different context. If the coefficient is dropped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A student is asked to simplify $9y - y$ and says $9y - y = 9$ (treating y as if it disappears). Is this correct?

Correct response: No: $9y - y = 8y$ ($y = 1y$, so $9y - 1y = 8y$). The variable does not disappear.

Teach: Read every bare variable out loud as '1x' before combining. Compare variable parts explicitly before adding coefficients.

Fresh Check (Recalculation Check)

Item: Simplify: $7y + y$

Correct response: $8y$.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A recipe uses 2 cups of flour plus 5 more cups of flour, written as an expression: $2c + 5c$. Simplify it, in a baking context with no algebra vocabulary used.

Correct response: $7c$.

Route: If check passes: move to AL-EX-02. If check fails: reteach reading bare variables as $1x$ on every item until it holds twice.

AL-EX-02 Distribute a negative sign across brackets

Mathematical claim: A negative sign in front of a bracket multiplies every term inside; two negatives in a row (subtracting a negative term) become a positive.

Route Finder

Item: Simplify: $5 - (x - 3)$

Correct response: $8 - x$ ($5 - x + 3 = 8 - x$).

Tripwire

Observed: A student simplifies $8 - (3x - 2)$ and writes $6 - 3x$.

Likely wrong response: $6 - 3x$, from distributing the negative to the first term but not the second, missing that $-1 \times -2 = +2$.

Correct response: $10 - 3x$ ($8 - 3x + 2 = 10 - 3x$).

Likely reasoning: may indicate the negative sign is being distributed to only the first term inside the bracket, not every term

Confirm (approach): Give a second bracket-distribution item with a different sign pattern inside. If a term is missed again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Simplify $12 - (5x - 7)$ and check your answer by substituting $x = 1$ into both the original and your simplified form.

Correct response: $19 - 5x$; at $x=1$: original = $12 - (5 - 7) = 12 - (-2) = 14$; simplified = $19 - 5 = 14$.
Confirmed.

Teach: Expand one term at a time, checking each sign separately against the original bracket before combining anything.

Fresh Check (Recalculation Check)

Item: Simplify: $10 - (4x - 5)$

Correct response: $15 - 4x$.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A budget note reads: starting balance 20, minus (a withdrawal of $6x$, minus a refund of 4). Write this as a simplified expression, with no mention of brackets or distribution.

Correct response: $24 - 6x$ ($20 - (6x - 4) = 20 - 6x + 4 = 24 - 6x$).

Route: If check passes: move to AL-EX-03. If check fails: reteach distributing term-by-term with signs checked individually.

AL-EX-03 Distinguish expressions from equations; translate words correctly

Mathematical claim: An expression has no equals sign and cannot be ‘solved,’ only simplified or evaluated; word phrases like ‘ n less than 5’ name an order that differs from reading left to right.

Route Finder

Item: Can ‘ $4x + 7$ ’ be solved for x ? Explain.

Correct response: No: it's an expression with no equals sign; it can only be simplified or evaluated at a given x .

Tripwire

Observed: A student translates ‘5 less than n ’ as $5 - n$.

Likely wrong response: $5 - n$, from translating the words in the exact order they appear.

Correct response: $n - 5$ ('less than' means take the first number away from the second).

Likely reasoning: *may indicate word-to-algebra translation is following literal word order rather than the operation's actual meaning*

Confirm (approach): Give a second 'less than' translation item with different numbers/variable. If the order is reversed again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Is ' $3x - 5 = 10$ ' an expression or an equation? Can it be 'solved'? What about ' $3x - 5$ '?

Correct response: $3x - 5 = 10$ is an equation and can be solved ($x=5$). $3x - 5$ alone is an expression and can only be simplified or evaluated.

Teach: Translate in two separate steps: name the operation first, then work out the order it applies in, rather than reading straight across.

Fresh Check (Recalculation Check)

Item: Translate: '8 less than m ', then simplify ' $2x + 6$ ' if $x = 3$.

Correct response: $m - 8$; and $2(3) + 6 = 12$.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A note says "7 less than twice a number," with no instruction to translate it in a particular order.

Correct response: $2n - 7$ (not $7 - 2n$).

Route: If check passes: chapter secure, move to AL-EX-04 (Chapter 2). If check fails: reteach the two-step translation method explicitly.

Chapter 2: Evaluating Algebraic Expressions

AL-EX-04 Substitute using order of operations, with brackets around negative values

Mathematical claim: Substituting a value means replacing the whole variable with that value, in brackets if negative, then following order of operations: not doing whichever operation looks easiest first.

Prerequisites: AL-EX-01

Route Finder

Item: Evaluate $3x + 2$ when $x = 4$.

Correct response: 14 ($3 \times 4 + 2 = 12 + 2 = 14$).

Tripwire

Observed: A student evaluates x^2 when $x = -5$ and writes -25.

Likely wrong response: -25, from writing -5^2 without brackets, which squares only the 5 and keeps the negative outside.

Correct response: 25 ($(-5)^2 = 25$: the entire value -5 is squared).

Likely reasoning: may indicate a negative value is being substituted without brackets, so the negative sign is left outside the operation instead of included in it

Confirm (approach): Give a second negative-substitution item with a different exponent or operation. If the bracket is dropped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Evaluate $3x^2$ when $x = -2$, showing the bracket step.

Correct response: 12 ($3 \times (-2)^2 = 3 \times 4 = 12$).

Teach: Require the substitution line written with brackets around every replaced value before any calculation begins, especially negatives.

Fresh Check (Recalculation Check)

Item: Evaluate $2x^2$ when $x = -3$.

Correct response: $18 (2 \times (-3)^2 = 2 \times 9 = 18)$.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A ball's height is given by $5 - t^2$ at t seconds. Find the height when $t = -3$, phrased as a physics question with no substitution instructions given.

Correct response: $-4 (5 - (-3)^2 = 5 - 9 = -4)$.

Route: If check passes: move to AL-EX-05. If check fails: reteach the bracketed-substitution line as a mandatory first step on every item.

AL-EX-05 Recognize an expression's value depends on input; interpret results in context

Mathematical claim: An expression like $2x + 1$ does not have one fixed value; it changes with x , and a numeric result in an application problem needs its units or meaning restated, not left as a bare number.

Prerequisites: AL-EX-04

Route Finder

Item: Find the value of $2x + 1$ when $x = 3$, and again when $x = 10$.

Correct response: 7, and 21: same expression, different inputs, different outputs.

Tripwire

Observed: A student evaluates a cost expression and reports the answer as just '35' with no units.

Likely wrong response: 35 alone, with the real-world meaning of the number dropped.

Correct response: \$35 (or whatever unit the expression represents): the numeric result must be restated in context.

Likely reasoning: may indicate the numeric answer is being reported without reconnecting it to what the expression actually represents

Confirm (approach): Give a second application item in a different context (distance, time, etc.), checking whether units are restated. If dropped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A taxi cost is $4 + 2m$ dollars for m minutes. Evaluate at $m = 10$ and $m = 20$. Is the cost fixed?

Correct response: \$24 and \$44: different inputs give different outputs; the expression's value is not fixed.

Teach: Require the answer restated in a full sentence naming what the number means, not just the bare value.

Fresh Check (Recalculation Check)

Item: A plumber's cost is $40 + 25h$ dollars for h hours. Evaluate at $h = 3$ and state the answer in context.

Correct response: \$115 for 3 hours of work.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A phone plan is described only as ' $10 + 0.05t$ ' with t being texts sent, and someone asks what the plan costs for 50 texts, with no units mentioned in the question.

Correct response: \$12.50 ($10 + 0.05 \times 50 = 10 + 2.50$), stated in dollars.

Route: If check passes: chapter secure, move to AL-EQ-01 (Chapter 3). If check fails: reteach requiring a context sentence on every application item.

Chapter 3: Solving One-Step Equations

AL-EQ-01 Solve a one-step equation using the correct inverse operation on both sides

Mathematical claim: Solving an equation requires applying the inverse of whatever operation is done to the variable, to both sides equally: not repeating the operation already shown, and not applying it to only one side.

Prerequisites: AL-EX-02

Route Finder

Item: Solve: $x + 5 = 12$

Correct response: $x = 7$ (subtract 5 from both sides).

Tripwire

Observed: A student solves $-3x = 12$ and writes $x = 4$.

Likely wrong response: $x = 4$, from dividing by 3 instead of -3, dropping the negative sign.

Correct response: $x = -4$ (divide both sides by -3, not 3).

Likely reasoning: *may indicate the negative sign on the coefficient is being dropped rather than divided through*

Confirm (approach): Give a second negative-coefficient equation with a different number. If the sign is dropped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Solve $-5x = -15$, and identify the inverse operation used.

Correct response: $x = 3$; divide both sides by -5.

Teach: Ask what operation undoes the one shown, including its sign, before touching either side of the equation.

Fresh Check (Recalculation Check)

Item: Solve: $-4x = 20$

Correct response: $x = -5$.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A scale shows a mystery weight plus 7 kg balances at 12 kg, written as $w + 7 = 12$ in a science context with no equation-solving instructions.

Correct response: $w = 5$ kg.

Route: If check passes: move to AL-EQ-02. If check fails: reteach identifying the exact inverse operation, including sign, before solving.

AL-EQ-02 Verify a solution by substituting back into the original equation

Mathematical claim: A solution should be checked by substituting it back into the original equation and confirming both sides are equal: stopping once a value for x appears is not the same as verifying it.

Prerequisites: AL-EQ-01

Route Finder

Item: You solved $x - 8 = 15$ and got $x = 23$. Verify.

Correct response: $23 - 8 = 15$. Confirmed.

Tripwire

Observed: A student solves $-3x = 12$ and gets $x = -4$, then stops without checking.

Likely wrong response: Stops at $x = -4$ with no verification step shown.

Correct response: Check: $-3(-4) = 12$. Confirmed. The check step should not be skipped, even when the sign work looked difficult.

Likely reasoning: may indicate checking is being treated as optional or extra, rather than the step that catches sign errors specifically

Confirm (approach): Give a second one-step equation and explicitly require the check shown. If it's skipped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A classmate solves $2x = -18$ and gets $x = -9$, then says checking is unnecessary since the steps were easy. Respond.

Correct response: Checking should still happen: $2 \times (-9) = -18$. Confirmed correct here, but the habit of skipping checks is risky regardless of how easy a problem feels.

Teach: Treat checking as mandatory on every equation with a negative or fraction involved, not just the ones that feel uncertain.

Fresh Check (Recalculation Check)

Item: Solve $x + 9 = 4$ and verify your answer.

Correct response: $x = -5$; check: $-5 + 9 = 4$. Confirmed.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: Someone calculates that a value of 6 solves $x - 10 = -4$, and wants to know if that's right, with no verification method suggested.

Correct response: Yes: $6 - 10 = -4$, confirmed by substitution.

Route: If check passes: chapter secure, move to AL-EQ-03 (Chapter 4). If check fails: reteach requiring the substitution check as a graded part of every answer.

Chapter 4: Solving Two-Step and Multi-Step Equations

AL-EQ-03 Solve a multi-step equation with brackets fully and correctly resolved

Mathematical claim: A multi-step equation with brackets can be solved by distributing first or by dividing by a shared factor first: either order is valid, provided every term inside the bracket is handled correctly, not just the first one.

Prerequisites: AL-EQ-01

Route Finder

Item: Solve: $2(x - 4) = 10$

Correct response: $x = 9$ (either distribute: $2x - 8 = 10$, $2x = 18$, $x = 9$; or divide by 2 first: $x - 4 = 5$, $x = 9$: both valid).

Tripwire

Observed: A student solves $2(x - 4) = 10$ by distributing and writes $2x - 4 = 10$.

Likely wrong response: $2x - 4 = 10$ ($x = 7$), from multiplying only the x -term by 2 and leaving the -4 unmultiplied.

Correct response: $2x - 8 = 10$, so $x = 9$.

Likely reasoning: may indicate the multiplier is being distributed to only one term inside the bracket, not both

Confirm (approach): Give a second bracket-equation item, checking whether both terms get distributed. If one is missed again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Solve $4(x - 2) = 20$ using both methods (distribute first, and divide by 4 first). Do they agree?

Correct response: $x = 7$ both ways: distribute: $4x - 8 = 20$, $4x = 28$, $x = 7$; divide first: $x - 2 = 5$, $x = 7$.

Teach: Whichever order is chosen (distribute or divide first), require every term in the bracket to be shown getting the same treatment before moving on.

Fresh Check (Recalculation Check)

Item: Solve: $3(x + 5) = 21$, showing your method.

Correct response: $x = 2$ (either method).

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A rectangle's perimeter is modeled as $2(x + 5) = 26$, presented as a geometry problem with no algebra-method instructions.

Correct response: $x = 8$ (either method).

Route: If check passes: move to AL-EQ-04. If check fails: reteach distributing to every term explicitly, one at a time.

AL-EQ-04 Recognize and interpret no-solution and infinitely-many-solutions cases

Mathematical claim: If solving an equation makes the variable disappear, the remaining statement decides the outcome: a false statement (like $2 = -3$) means no solution; a true statement (like $4 = 4$) means infinitely many solutions: not an error to keep solving through.

Prerequisites: AL-EQ-03

Route Finder

Item: Solve: $2x + 3 = 2x + 3$

Correct response: Infinitely many solutions (the statement $3 = 3$ is always true).

Tripwire

Observed: A student solves an equation, the variables cancel, and they reach $2 = -3$. The student says they must have made an arithmetic mistake and keeps re-solving.

Likely wrong response: Continues trying to re-solve, assuming every equation must produce a value for x .

Correct response: No mistake: $2 = -3$ is false, which means the equation has no solution.

Likely reasoning: *may indicate the disappearance of the variable is being treated as an error rather than a meaningful result to interpret*

Confirm (approach): Give a second equation that leads to a true statement (infinitely many solutions) rather than false, and check whether it's correctly distinguished. If misinterpreted again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Solve $5x - 2 = 5x + 3$. What does the result mean?

Correct response: $-2 = 3$, false: no solution.

Teach: When the variable disappears, stop and evaluate the remaining numeric statement as true or false: that's the answer, not a sign to keep working.

Fresh Check (Recalculation Check)

Item: Solve: $3(x + 2) = 3x + 6$. What does the result mean?

Correct response: $6 = 6$, true: infinitely many solutions.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: Two phone plans are modeled as $20 + 0.1m$ and $20 + 0.1m$ for m minutes, and someone asks when the plans cost the same, with no mention of equation types.

Correct response: Always: the two expressions are identical, so every value of m works (infinitely many solutions).

Route: If check passes: chapter secure, move to AL-IN-01 (Chapter 5). If check fails: reteach the true/false interpretation step explicitly.

Chapter 5: Inequalities**AL-IN-01 Flip the inequality sign when multiplying or dividing by a negative**

Mathematical claim: Solving an inequality follows the same steps as an equation, except that multiplying or dividing both sides by a negative number reverses the direction of the inequality sign.

Prerequisites: AL-EQ-01

Route Finder

Item: Solve: $-2x > 8$

Correct response: $x < -4$ (divide by -2, flip the sign).

Tripwire

Observed: A student solves $-4x > 20$ and writes $x > -5$.

Likely wrong response: $x > -5$, from dividing by the negative without flipping the inequality direction.

Correct response: $x < -5$ (dividing by -4 requires flipping the sign).

Likely reasoning: may indicate a negative divisor is being treated the same as a positive one, without flipping the inequality

Confirm (approach): Give a second negative-coefficient inequality with a different sign direction. If the flip is missed again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Solve $-5x \geq -25$, and state whether the sign flips.

Correct response: $x \leq 5$; dividing by a negative flips the sign from $\geq$ to $\leq$.

Teach: Before dividing or multiplying, ask whether the number is negative: if so, say the flip out loud as part of the step, not after.

Fresh Check (Recalculation Check)

Item: Solve: $-3x \leq 15$

Correct response: $x \geq -5$.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A note says a temperature change of -2 degrees per hour times some number of hours is at most -10 degrees total, written as $-2h \leq -10$ with no inequality-solving instructions.

Correct response: $h \geq 5$ (dividing by -2 flips the sign).

Route: If check passes: move to AL-IN-02. If check fails: reteach the flip rule as a checked step every time a negative is involved.

AL-IN-02 Graph an inequality correctly and interpret the solution set in context

Mathematical claim: A strict inequality ($<$ or $>$) uses an open circle at the boundary; 'or equal to' ($\leq$ or $\geq$) uses a closed circle; the arrow points toward the values that make the inequality true, not toward a fixed direction based on the symbol's shape. In an application, a non-whole-number boundary must be rounded to a sensible whole value.

Prerequisites: AL-IN-01

Route Finder

Item: Graph $x \geq 5$ in words: open or closed circle, which direction?

Correct response: Closed circle at 5, arrow pointing right (values 5 and greater).

Tripwire

Observed: A student graphs $x < -2$ with the arrow pointing to the right.

Likely wrong response: Arrow pointing right, away from the direction the values actually lie.

Correct response: Open circle at -2, arrow pointing left (values less than -2 are to the left on a number line).

Likely reasoning: *may indicate the arrow direction is being chosen without checking which values actually satisfy the inequality*

Confirm (approach): Give a second inequality with the opposite direction, checking whether the arrow is placed by reasoning about values, not habit. If reversed again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Graph $x \leq -3$ in words, and state one value that should be included and one that shouldn't.

Correct response: Closed circle at -3, arrow left; -3 and -4 are included, -2 is not.

Teach: Before drawing the arrow, name one value that should be included and check which side of the boundary it falls on.

Fresh Check (Recalculation Check)

Item: A ticket costs \$12.50 each. A group has \$500. Solve for how many tickets they can buy, and interpret the result.

Correct response: $p \leq 40$; since ticket count must be a whole number, at most 40 tickets.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A parking garage charges by the hour, and a sign states a driver can stay for at most 6 hours, modeled as $h \leq 6$, with no graphing instructions given.

Correct response: Closed circle at 6, arrow pointing left (values 6 and below).

Route: If check passes: chapter secure, move to AL-LN-01 (Chapter 6). If check fails: reteach checking a test value before drawing the graph.

Chapter 6: Graphing Linear Equations and Slope

AL-LN-01 Identify slope and y-intercept, including implicit coefficients

Mathematical claim: In $y = mx + b$, m is the slope and b is the y-intercept; when x has no visible coefficient, the coefficient is -1 or 1, not 0.

Route Finder

Item: Identify the slope and y-intercept of $y = 3x - 2$.

Correct response: $m = 3$, $b = -2$.

Tripwire

Observed: A student is asked for the slope of $y = -x + 4$ and says the slope is 0 because no number is written in front of x .

Likely wrong response: Slope 0, from treating the invisible coefficient as absent rather than -1.

Correct response: $m = -1$ (since $-x = -1x$), $b = 4$.

Likely reasoning: may indicate an invisible coefficient on x is being read as 0 rather than ± 1

Confirm (approach): Give a second equation with an implicit +1 coefficient (e.g., $y = x + 5$), checking whether it's correctly read as slope 1. If misread again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Identify the slope and y-intercept of $y = x - 9$.

Correct response: $m = 1$, $b = -9$.

Teach: Rewrite $-x$ or x explicitly as $-1x$ or $1x$ before naming the slope, the same way a bare variable was handled in Chapter 1.

Fresh Check (Recalculation Check)

Item: Identify the slope and y-intercept of $y = -x - 7$.

Correct response: $m = -1$, $b = -7$.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A cost model is written as $C = -n + 50$ with no slope/intercept language used, and someone asks how the cost changes per unit.

Correct response: Decreases by 1 per unit (slope -1); starts at 50 (y-intercept 50).

Route: If check passes: move to AL-LN-02. If check fails: reteach rewriting implicit coefficients explicitly before reading slope.

AL-LN-02 Calculate slope from two points; distinguish zero slope from undefined slope

Mathematical claim: Slope is vertical change over horizontal change, computed with the two points' coordinates in consistent order; a horizontal line has slope 0, while a vertical line has undefined slope (division by 0), and the two are not the same thing.

Prerequisites: AL-LN-01

Route Finder

Item: Find the slope through (1, 2) and (5, 10).

Correct response: $m = 2 \left(\frac{10-2}{5-1} = \frac{8}{4} = 2 \right)$.

Tripwire

Observed: A student is shown a horizontal line and a vertical line and says both have slope 0.

Likely wrong response: Both lines called slope 0, from equating 'not slanted' with 'slope 0' regardless of which direction it isn't slanted in.

Correct response: The horizontal line has slope 0; the vertical line has undefined slope (the run is 0, and division by 0 is undefined).

Likely reasoning: *may indicate zero slope and undefined slope are being treated as the same thing because neither line looks slanted*

Confirm (approach): Give a second pair of one horizontal and one vertical line's coordinates, checking whether the distinction holds. If conflated again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Find the slope through (0, 4) and (0, -2). What kind of line is this?

Correct response: Undefined slope (run = 0); this is a vertical line.

Teach: Compute the actual rise and run numerically for each line rather than judging by appearance; division by 0 is the flag for undefined, not a slanted look.

Fresh Check (Recalculation Check)

Item: Find the slope through (3, 1) and (3, 8), and through (2, 5) and (6, 5).

Correct response: First pair: undefined (vertical line, run = 0). Second pair: 0 (horizontal line, rise = 0).

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: Two delivery routes are compared using coordinate pairs (1,3)-(1,9) and (2,4)-(8,4), with no mention of slope in the question.

Correct response: First: undefined slope (vertical). Second: slope 0 (horizontal).

Route: If check passes: chapter secure, move to AL-LN-03 (Chapter 7). If check fails: reteach computing $\frac{\text{rise}}{\text{run}}$ numerically before naming the slope type.

Chapter 7: Writing Linear Equations**AL-LN-03 Use point-slope form correctly with negative coordinates**

Mathematical claim: Point-slope form $y - y_1 = m(x - x_1)$ requires substituting the point's coordinates exactly as given, including their signs; subtracting a negative y-coordinate becomes adding, and dropping that sign changes the equation.

Prerequisites: AL-LN-02

Route Finder

Item: Write the equation of a line with slope 4 through the point (1, 5).

Correct response: $y - 5 = 4(x - 1)$.

Tripwire

Observed: A student writes the point-slope equation for a line with slope 5 through (2, -3) as $y - 3 = 5(x - 2)$.

Likely wrong response: $y - 3 = 5(x - 2)$, from changing -3 into +3 in the wrong spot instead of correctly resolving $y - (-3)$.

Correct response: $y + 3 = 5(x - 2)$, since $y - (-3) = y + 3$.

Likely reasoning: may indicate a negative y-coordinate is being dropped or its sign moved incorrectly rather than resolved as subtracting a negative

Confirm (approach): Give a second point-slope item with a negative x-coordinate instead, checking the same sign-handling. If mishandled again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Write the point-slope equation for a line with slope -3 through (-2, -5), then simplify.

Correct response: $y + 5 = -3(x + 2)$; simplifies to $y = -3x - 11$.

Teach: Write the substitution with the coordinate's original sign kept visible in brackets before simplifying, the same discipline as bracketed negative substitution in Chapter 2.

Fresh Check (Recalculation Check)

Item: Write the point-slope equation for a line with slope -2 through (-4, 6), then simplify to slope-intercept form.

Correct response: $y - 6 = -2(x + 4)$; simplifies to $y = -2x - 2$.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A line on a graph passes through the point (5, -1) and rises 2 units for every 1 unit right, phrased with no mention of slope or point-slope form.

Correct response: $y + 1 = 2(x - 5)$, which simplifies to $y = 2x - 11$.

Route: If check passes: move to AL-LN-04. If check fails: reteach keeping coordinate signs visible in brackets before simplifying.

AL-LN-04 Find a perpendicular slope; recognize vertical lines can't be written in $y=mx+b$

Mathematical claim: A perpendicular slope is the negative reciprocal of the original slope, not just the reciprocal; a vertical line has undefined slope and must be written as $x = a$ constant, never forced into $y = mx + b$.

Prerequisites: AL-LN-02

Route Finder

Item: A line has slope 3. What is the slope of a line perpendicular to it?

Correct response: $-\frac{1}{3}$ (negative reciprocal).

Tripwire

Observed: A student is told a line has slope 4 and says the perpendicular slope is $\frac{1}{4}$.

Likely wrong response: $\frac{1}{4}$, from taking the reciprocal but forgetting the negative sign.

Correct response: $-\frac{1}{4}$ (the negative reciprocal, since $4 \times (-\frac{1}{4}) = -1$).

Likely reasoning: may indicate the reciprocal is being taken without applying the required negative sign for perpendicularity

Confirm (approach): Give a second perpendicular-slope item with a fractional original slope, checking whether the negative sign is retained. If dropped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: A line has slope $-\frac{5}{6}$. Find the perpendicular slope, and verify using the product rule.

Correct response: $\frac{6}{5}$; check: $(-\frac{5}{6}) \times (\frac{6}{5}) = -1$. Confirmed.

Teach: Require the check $m_1 \times m_2 = -1$ shown explicitly before accepting a perpendicular slope as correct.

Fresh Check (Recalculation Check)

Item: A line has slope $-\frac{2}{5}$. Find the perpendicular slope, and state the equation of the vertical line through (7, 3).

Correct response: $\frac{5}{2}$; and $x = 7$ (a vertical line cannot be written in $y = mx + b$).

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: Two hiking trails cross at a right angle; one has a steepness ratio of 3 (rise over run), and someone asks for the other trail's steepness ratio, with no perpendicular-slope language used.

Correct response: $-\frac{1}{3}$.

Route: If check passes: chapter secure, move to AL-SY-01 (Chapter 8). If check fails: reteach the $m_1 \times m_2 = -1$ verification step.

Chapter 8: Systems of Equations

AL-SY-01 Solve a system with correct bracket and sign handling

Mathematical claim: Substitution requires the entire replaced expression to be enclosed in brackets before distributing; elimination requires the negative sign to be distributed across every term of the equation being subtracted, not just the first.

Prerequisites: AL-EQ-03

Route Finder

Item: Solve by substitution: $y = x - 3$ and $2x - y = 7$

Correct response: $x = 4, y = 1$ ($2x - (x-3) = 7, x + 3 = 7, x = 4, y = 1$).

Tripwire

Observed: A student substitutes $y = x - 3$ into $2x - 4y = 10$ and writes $2x - 4x - 3 = 10$.

Likely wrong response: $2x - 4x - 3 = 10$, from substituting without enclosing the whole replaced expression in brackets.

Correct response: $2x - 4(x - 3) = 10$, which becomes $2x - 4x + 12 = 10$.

Likely reasoning: *may indicate the substituted expression is being inserted without brackets, so only part of it gets the intended operation applied*

Confirm (approach): Give a second substitution item where the substituted expression has two terms, checking whether brackets are used. If dropped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Solve by elimination: $3x + y = 11$ and $x - y = 1$.

Correct response: $x = 3, y = 2$.

Teach: Require the substituted expression written in brackets as an explicit, visible step before any distribution happens.

Fresh Check (Recalculation Check)

Item: Solve by substitution: $y = 2x + 1$ and $3x - 2y = -8$

Correct response: $x = 6, y = 13$.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: Two friends' combined ages and age difference are given as $x + y = 30$ and $x - y = 4$, in a word-problem context with no method specified.

Correct response: $x = 17, y = 13$.

Route: If check passes: move to AL-SY-02. If check fails: reteach the bracket-the-whole-substitution rule explicitly.

AL-SY-02 Interpret the result of solving a system

Mathematical claim: A system's solution is an ordered pair naming both variables, not a single value; if solving leads to a false numeric statement, the lines are parallel with no solution, and if it leads to a true statement, the lines coincide with infinitely many solutions.

Prerequisites: AL-SY-01

Route Finder

Item: Solve the system $x + y = 7$ and $x - y = 3$, and state the solution.

Correct response: (5, 2): an ordered pair, not just $x = 5$.

Tripwire

Observed: A student solves a system, reaches $14 = 15$, and says they must have made an arithmetic mistake because systems always have one answer.

Likely wrong response: Assumes an error was made and keeps re-solving, rather than interpreting the false statement.

Correct response: No mistake: $14 = 15$ is false, meaning the system has no solution (the lines are parallel).

Likely reasoning: *may indicate every system is being assumed to have exactly one solution, rather than checking what a contradictory result actually means*

Confirm (approach): Give a second system that leads to a true statement instead (infinitely many solutions), checking whether it's distinguished from the no-solution case. If confused again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Solve the system $x - y = 4$ and $2x - 2y = 9$. What does the result mean?

Correct response: Substituting shows $8 = 9$, false: no solution (parallel lines).

Teach: When variables disappear during solving, evaluate the remaining statement as true or false, the same interpretation step used for single equations in Chapter 4.

Fresh Check (Recalculation Check)

Item: Solve the system $2x + y = 5$ and $4x + 2y = 10$. What does the result mean?

Correct response: $10 = 10$, true: infinitely many solutions (the equations describe the same line).

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: Two pricing plans are compared and the algebra leads to $6 = 6$ with both variables gone, in a business context with no system-of-equations language used.

Correct response: True statement: infinitely many solutions; the two plans are identical.

Route: If check passes: chapter secure, move to AL-PL-01 (Chapter 9). If check fails: reteach the true/false interpretation, cross-referencing AL-EQ-04.

Chapter 9: Polynomials: Operations

AL-PL-01 Add and subtract polynomials, distributing the negative fully

Mathematical claim: Subtracting a polynomial means distributing a negative sign across every one of its terms, not just the first; only genuinely like terms (same variable and exponent) combine.

Prerequisites: AL-EX-02

Route Finder

Item: Simplify: $(5x^2 + 3x) - (2x^2 + x)$

Correct response: $3x^2 + 2x$.

Tripwire

Observed: A student simplifies $(4x^2 - 3x) - (x^2 + 5x)$ and writes $3x^2 + 2x$.

Likely wrong response: $3x^2 + 2x$, from subtracting the first term correctly but adding the second instead of subtracting it.

Correct response: $3x^2 - 8x$ (distribute the negative across both terms: $4x^2 - 3x - x^2 - 5x = 3x^2 - 8x$).

Likely reasoning: may indicate the negative sign is being distributed to only the first term of the second polynomial, not every term

Confirm (approach): Give a second polynomial-subtraction item with three terms in the second polynomial, checking full distribution. If a term is missed again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Simplify: $(2x^2 - x + 6) - (5x^2 - 3x - 2)$

Correct response: $-3x^2 + 2x + 8$.

Teach: Rewrite the subtraction as adding the opposite of every term first, the same discipline as bracket distribution in Chapter 1.

Fresh Check (Recalculation Check)

Item: Simplify: $(7x^2 + 2x - 4) - (3x^2 - 5x + 1)$

Correct response: $4x^2 + 7x - 5$.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: Two account balances are modeled as polynomials, and someone asks for the difference between them, in a finance context with no polynomial-subtraction instructions given.

Correct response: Any correctly distributed and combined result, provided the negative is applied to every term of the second polynomial.

Route: If check passes: move to AL-PL-02. If check fails: reteach distributing the negative across every term, cross-referencing AL-EX-02.

AL-PL-02 Multiply binomials with full distribution

Mathematical claim: Multiplying two binomials requires every term in the first to multiply every term in the second (four products total), including the two middle

products that combine into the middle term of the result; multiplying variables multiplies their exponents, it does not add the variables' values.

Prerequisites: AL-PL-01

Route Finder

Item: Expand: $(x + 3)(x + 4)$

Correct response: $x^2 + 7x + 12$.

Tripwire

Observed: A student expands $(x + 5)^2$ and writes $x^2 + 25$.

Likely wrong response: $x^2 + 25$, from squaring each term separately and skipping the two middle products entirely.

Correct response: $x^2 + 10x + 25$ ($(x+5)(x+5) = x^2 + 5x + 5x + 25$).

Likely reasoning: may indicate a squared binomial is being expanded by squaring each term individually, skipping the middle term produced by cross-multiplication

Confirm (approach): Give a second squared-binomial item with different numbers, checking whether the middle term appears. If skipped again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Expand $(x - 3)(x + 8)$, showing all four products.

Correct response: $x^2 + 5x - 24$.

Teach: Write out all four products explicitly (first, outer, inner, last) before combining, even for a squared binomial written as a product twice.

Fresh Check (Recalculation Check)

Item: Expand: $(x - 6)^2$

Correct response: $x^2 - 12x + 36$.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A garden's area is modeled as $(x + 4)$ times $(x + 4)$ for a square plot, with no binomial-expansion language used.

Correct response: $x^2 + 8x + 16$.

Route: If check passes: chapter secure, move to AL-PL-03 (Chapter 10). If check fails: reteach writing all four products explicitly before combining.

Chapter 10: Factoring

AL-PL-03 Check for a GCF before factoring the remaining trinomial

Mathematical claim: Factoring should always start by checking for a greatest common factor across all terms; factoring only the trinomial pattern inside while ignoring a shared numeric factor leaves the expression incompletely factored.

Prerequisites: AL-PL-02

Route Finder

Item: Factor completely: $2x^2 + 10x + 12$

Correct response: $2(x + 2)(x + 3)$ (GCF of 2 first, then factor the trinomial).

Tripwire

Observed: A student factors $2x^2 + 10x + 12$ and writes $(x + 2)(x + 3)$, saying they found two numbers that multiply to 6 and add to 5.

Likely wrong response: $(x + 2)(x + 3)$, missing the GCF of 2 that should have been factored out before the trinomial.

Correct response: $2(x + 2)(x + 3)$: the trinomial factoring is correct, but the GCF of 2 was never factored out first.

Likely reasoning: *may indicate the GCF step is being skipped, going straight to trinomial factoring without checking for a shared factor first*

Confirm (approach): Give a second trinomial with a GCF other than 2, checking whether it's caught before the trinomial factoring. If missed again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Factor completely: $4x^2 + 20x + 24$

Correct response: $4(x + 2)(x + 3)$.

Teach: Require a GCF check as the explicit first line of work on every factoring problem, before any other method is attempted.

Fresh Check (Recalculation Check)

Item: Factor completely: $3x^2 + 15x + 18$

Correct response: $3(x + 2)(x + 3)$.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: An area model shows $5x^2 + 25x + 30$ square units and asks for its factored dimensions, with no GCF language used.

Correct response: $5(x + 2)(x + 3)$.

Route: If check passes: move to AL-PL-04. If check fails: reteach the GCF-first rule as a mandatory first step, no exceptions.

AL-PL-04 Verify a factored form by expanding; recognize non-factorable sums of squares

Mathematical claim: A factored form can be checked by expanding it back to confirm it matches the original polynomial; a sum of two squares (like $x^2 + 25$) does not factor over the reals using the difference-of-squares pattern, since that pattern requires a subtraction, not an addition.

Prerequisites: AL-PL-03

Route Finder

Item: Factor $x^2 - 16$, then verify by expanding.

Correct response: $(x + 4)(x - 4)$; expanding gives $x^2 - 16$, confirmed.

Tripwire

Observed: A student factors $x^2 + 25$ as $(x + 5)(x - 5)$.

Likely wrong response: $(x + 5)(x - 5)$, applying the difference-of-squares pattern to an expression that is a sum, not a difference.

Correct response: $x^2 + 25$ does not factor over the reals: $(x+5)(x-5)$ expands to $x^2 - 25$, not $x^2 + 25$, so the pattern doesn't apply to a sum.

Likely reasoning: *may indicate the difference-of-squares pattern is being applied whenever two perfect squares are seen, without checking whether they're being added or subtracted*

Confirm (approach): Give a second item mixing a genuine difference of squares with a sum of squares, checking whether both are handled correctly. If the sum is force-factored again, treat as confirmed.

Confirmation Item

Same claim, different surface form.

Item: Factor $x^2 - 100$ if possible, then check by expanding. Can $x^2 + 100$ be factored the same way?

Correct response: $(x+10)(x-10)$, confirmed by expanding. $x^2 + 100$ cannot use this pattern since it is a sum, not a difference.

Teach: Require expansion-verification on every factored answer; a sum of squares that doesn't expand back correctly is the signal that it doesn't factor this way.

Fresh Check (Recalculation Check)

Item: Factor $x^2 - 49$ if possible, and explain whether $x^2 + 49$ can be factored the same way.

Correct response: $x^2 - 49 = (x+7)(x-7)$; $x^2 + 49$ cannot be factored this way since it's a sum, not a difference.

Road Test

Less-cued, mixed context: tests whether the skill transfers without being told which chapter this is from.

Item: A student claims two square rooms, one $x^2 - 36$ and one $x^2 + 36$ in area, can both be split into equal factor pairs the same way, with no sum/difference vocabulary used.

Correct response: Only $x^2 - 36$ factors this way, as $(x+6)(x-6)$; $x^2 + 36$ does not factor over the reals using this pattern.

Route: Chapter 10 and the full Book 2 skill map are complete once both AL-PL-03 and AL-PL-04 are secure. Cross-book route: Book 4 continues this series once built.